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Compressive Sampling

One of the central tenets of signal processing and data acquisition is the Shannon/Nyquist sampling theory, which states that the number of samples required to capture a signal is dictated by its bandwidth. It is not an overstatement to say that this theory underlies most sensing, signal acquisition, and analog-to-digital conversion protocols in use today. However, it is well known that the Nyquist rate is a sufficient, but by no means necessary, condition. Over the last few years, an alternative sampling/sensing theory, known as "compressive sampling" or "compressed sensing" (CS), enables the faithful recovery of signals, images, and other data from what appear to be highly sub-Nyquist-rate samples. How is this possible?

At the heart of the new approach are two crucial observations. The first is that the Shannon/Nyquist signal representation exploits only minimal prior knowledge about the signal being sampled, namely its bandwidth. However, most objects we are interested in acquiring are structured and depend upon a smaller number of degrees of freedom than the bandwidth suggests. In other words, most objects of interest are *sparse* or *compressible* in the sense that they can be encoded with just a few numbers without much numerical or perceptual loss.

The second observation is that the useful information content in compressible signals can be captured via sampling or sensing protocols that directly condense signals into a small amount of data. A surprise is that many such sensing protocols do nothing more than linearly correlate the signal with a fixed set of signal-independent waveforms. These waveforms, however, need to be "incoherent" with the family of waveforms in which the signal is compressible. One then typically uses numerical optimization to reconstruct the signal from the linear measurements.

[THE THEORY OF CS ASSERTS THAT ONE CAN COMBINE "LOW-RATE SAMPLING" WITH COMPUTATIONAL POWER FOR EFFICIENT AND ACCURATE SIGNAL ACQUISITION.]

In short, and in stark contrast with conventional wisdom, the theory of CS asserts that one can combine "low-rate sampling" with computational power for efficient and accurate signal acquisition. This point of view is at once simple and powerful: CS bypasses the current, often wasteful, acquisition process in which massive amounts of data are collected only to be—in large part—discarded by a subsequent compression stage, which is usually necessary for storage and transmission purposes. CS data acquisition systems directly translate analog data into a compressed digital form so that one can—at least in principle—obtain super-resolved signals from just a few measurements. After the acquisition step, all we need to do is "decompress" the measured data through an optimization.

Retrieving information from signals with intrinsically few degrees of freedom is a classic parameter estimation problem, going back as far Prony's method of the late 18th century. In modern signal processing, the retrieval of sinusoids buried in noise has led to a rich literature over the last three decades. Posing this as a sampling problem for classes of nonbandlimited but compressible signals is more recent, as evidenced by the recent development of sampling finite-rate-of-innovation signals (i.e., signals with a finite number of degrees of freedom per unit time). In this case, structured, deterministic sampling kernels are used, allowing continuous-time sampling at twice the signal's "innovation rate" rather than twice its bandwidth. CS employs a more general approach and is typically based on random kernels and nonparametric estimation techniques. CS is very broadly applicable and enables the recovery of any compressible signal from very few samples. The estimation framework is based on solving an underdetermined linear system of equations with a compressible or sparse unknown.

This special section aims to present the key ideas underlying the new CS theory as well as selected applications areas where the theory promises to have a significant impact. With this in mind, the articles have been selected to provide the reader with specific insights into the basic theory, capabilities, and limitations of CS. In addition, we hope that the application articles will inspire some readers to develop their own novel applications.

On the theory side, six articles overview the current state-of-the-art in CS and related areas. Candès and Wakin survey some foundational CS results, showing that sparse signals can be recovered perfectly from just a

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few incoherent measurements. They also review the use of randomness to design good sensing mechanisms together with CS protocols that support robust signal reconstruction from noisy or quantized sensor data. Romberg argues that the incoherence property is akin to the existence of an uncertainty principle between the sparsity domain and the measurement domain, which generalizes the celebrated Weyl-Heisenberg uncertainty relation between time and frequency. It is precisely this principle that enables subsampling without information loss. Blu et al. consider the sampling of continuous-time signals that are not bandlimited but have a finite rate of innovation. Interestingly, one can design sampling kernels that enable perfect reconstruction of such signals provided that the sampling rate is above twice the rate of innovation. They also study the noisy case and provide performance bounds as well as practical algorithms for reaching these bounds. Lu and Do provide a geometrical interpretation of finite rate of innovation sampling and draw connections with randomized measurement techniques by modeling the signal being acquired as coming from a union of subspaces. Goyal, Fletcher, and Rangan explore the implications of CS for lossy compression and some of its relationships with universal source coding. Finally, CS is connected with exciting recent work in theoretical computer science and randomized algorithms. Gilbert et al. outline these connections and present a randomized algorithm to rapidly approximate the discrete Fourier transform by minimally sampling the digital input in the time domain. Beyond the obvious relationship with CS, an interesting corollary is the existence of approximate Fourier transform algorithms that are in some cases faster than the fast Fourier transform (FFT).

On the applications side, four articles survey the many ongoing efforts to build a new generation of sensing devices based on CS. Lustig et al. show how CS can help reduce the scan time in magnetic resonance imaging (MRI) and offer sharper images of living tissues. This is especially important because time-consuming MRI scans have traditionally limited the use of this sensing modality in important applications. Simply put, faster imaging here means novel applications. Duarte et al. survey a single-pixel digital camera based on a digital micromirror device that randomly modulates the light from the scene under view to compute random CS measurements. This enables simpler, smaller, and cheaper digital cameras that can operate efficiently across a much broader spectral range than conventional silicon-based cameras. Healy and Brady survey a new breed of optical devices that digitize holographic measurements instead of simple pixel samples. They also argue that CS can help address the enormous challenge of acquiring and processing ultrawideband radio frequency signals. Here CS enables the design of novel analog-to-digital converter architectures that exploit signal compressibility in order to dramatically reduce the sampling rate. Finally, Haupt et al. overview some of the ongoing work in CS for the decentralized compression and transmission of data collected by a network of sensors and show that the communication costs required to achieve a target distortion level at a desired receiver can be far less than the costs of conventional methods.

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